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SECOND-ORDER PERTURBATIONS IN ALFVÉN WAVES IN FINITE PRESSURE PLASMA

I.S. Dmitrienko
Institute of Solar-Terrestrial Physics SB RAS, Irkutsk, Russia, dmitrien@iszf.irk.ru

Abstract. It is shown first that in finite pressure plasma, just as in cold plasma, Alfvén waves created by an initial perturbation generate plasma flows and decreases in the magnetic field, which propagate along with these waves. Second, at the stage of their interaction, Alfvén waves generate slow magnetosonic (SMS) waves propagating along the magnetic field. These results suggest that at least some of the fast plasma flows observed in the magnetotail can be one of the manifestations of propagating Alfvén waves both in the magnetosphere regions with cold plasma and in the magnetosphere regions with finite pressure plasma. They also provide potential possibility for determining the position of a source of Alfvén disturbance from observations of Alfvén waves and their induced SMS waves.

Keywords: Alfvén waves, fast plasma flows, SMS waves.

INTRODUCTION

In this paper, generation and propagation of Alfvén waves in finite pressure plasma are analyzed to describe second-order effects caused by the interaction between these waves, considered as first-order perturbations. Such effects include the formation of longitudinal plasma flows by Alfvén waves and the generation of slow magnetosonic (SMS) waves. These effects may be fairly essential for understanding magnetospheric phenomena associated with Alfvén waves, primarily in terms of interpretation of plasma flows in the magnetotail [Zong et al., 2007; Fruhauff, Glassmeier, 2016; Takada et al., 2005; Raj et al., 2002; Cao et al.., 2006; Du et al., 2011; Zhang et al., 2020]. Alfvén waves are often observed in the magnetotail [Keiling et al., 2000, 2005; Keiling, 2009; Takada et al., 2006]. Alfvén perturbations in the magnetotail lobes and adjacent plasma sheet boundary layers (PSBL) can be generated both by fast magnetosonic (FMS) perturbations penetrating from the solar wind and by instabilities of different types, including reconnection in the magnetotail [Lee, 1998; Leonovich et al., 2003; Walker, 2005; Mazur, Chuiko, 2013; Klimushkin et al., 2012; Mager, Klimushkin, 2017; Birn et al., 2015; Leonovich, Kozlov, 2013]. Alfvén waves can also be generated by conversion of FMS modes of the magnetotail waveguide [Wright, Allan, 2008; Mazur et al., 2010; Dmitrienko, 2013]. At the same time, in the magnetotail there are so-called fast plasma flows propagating, as Alfvén waves do, both toward and from Earth, depending on the observation point. Magnetic field disturbances are usually observed in the region of existence of these flows.

Second-order perturbations in Alfvén waves have been studied in [Dmitrienko, 2019], using a magnetohydrodynamic model of cold plasma. Dmitrienko [2019] has shown that plasma flows along the magnetic field, which can be used to interpret plasma flows in the magnetotail, are formed in propagating Alfvén waves; it has also been demonstrated that a magnetic field disturbance oppositely directed to the undisturbed magnetic field is generated in a propagating Alfvén wave; this can be interpreted as a local decrease in intensity of the undisturbed magnetic field.

Note that the formation of flows in Alfvén waves has also been considered as a result of the development of secondary instability caused by nonlinear effects (see, e.g., [Pokhotelov et al., 2003, 2004; Zhao et al., 2012]). The authors pointed out a crucial role of kinetic effects in generating convective motions. In [Dmitrienko, 2019] and in this paper, the existence of flows caused by nonlinear effects is shown in the magnetohydrodynamic approximation. This difference between the results obtained in [Pokhotelov et al., 2003, 2004; Zhao et al., 2012] and the results of [Dmitrienko, 2019] and the present work is due to the difference in the formulation of problems, which is explained by the orientation to different physical phenomena.

Pokhotelov et al. [2003, 2004] and Zhao et al. [2012] suggest that the results can be applied to stationary turbulent processes, whereas [Dmitrienko, 2019] and this paper are aimed at applying them to perturbations localized in space and time. Accordingly, in [Pokhotelov et al., 2003, 2004; Zhao et al., 2012] convective flows are selected in terms of harmonic decomposition as a zero spatial harmonic with respect to the coordinate along the field — through appropriate averaging. In this case, the averaged longitudinal ponderomotive force exists only due to minor effects — dissipative [Dmitrienko, 1997, 1999, 2007], non-stationary (i.e. taking into account the low-frequency part of perturbation) [Dmitrienko, 2011] or kinetic [Pokhotelov et al., 2003, 2004; Zhao et al., 2012]. This paper deals with longitudinally localized Alfvén perturbations and hence with second-order flows also longitudinally localized. The longitudinal ponderomotive force generating such flows also exists in the absence of the minor effects, both kinetic and dissipative. As a result, it does not contain an additional smallness caused by these effects and can play a significant role in the phenomena associated with Alfvén waves.

A limitation of the applicability of the results received in [Dmitrienko, 2019] to the interpretation of magnetospheric phenomena is the cold plasma model used to obtain the results. It is applicable only to magnetotail lobes, whereas fast plasma flows are observed mainly in the magnetospheric regions projected into PSBL adjacent to the magnetotail lobes. In these regions, plasma has in fact a finite pressure. Since perturbations reaching Earth from distant regions of the magnetotail are of particular interest, transferring the study of second-order effects to the case of finite pressure plasma, we restrict ourselves to second-order perturbations propagating along the magnetic field.

1. INITIAL EQUATIONS

Assume that under undisturbed conditions plasma is at rest and its state is characterized by the following uniform parameters: $\mathbf{B}_0$ is the magnetic field, ρ_0, P_0 is the plasma density and pressure. Accordingly, denote the plasma perturbation: the magnetic field as $\mathbf{B}$, the plasma density, pressure, and velocity as ρ, P, and $\mathbf{v}$ respectively. Use the Cartesian coordinate system x, y, z. Let the magnetic field $\mathbf{B}_0$ be directed along the Z-axis.

Perturbations of the x and z components of the field and velocity, as well as pressure, density, and magnetic pressure, are assumed as small quantities of the same order. Leaving only the largest terms in the MHD equations for perturbations, obtain two sets of equations

$$\begin{aligned} &\partial_t B_y = B_0 \partial_z v_y, \\ &\rho_0 \partial_t v_y = \frac{1}{4\pi} B_0 \partial_z B_y \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\partial_t B_x = B_0 \partial_z v_x, \\ &\partial_t B_z = -B_0 \partial_x v_x, \\ &\rho_0 \partial_t v_x = -\partial_x P + \frac{B_0}{4\pi}\left(\partial_z B_x - \partial_x B_z\right) - \frac{1}{8\pi}\partial_x B_y^2, \\ &\rho_0 \partial_t v_z = -\partial_z \left(P + \frac{1}{8\pi} B_y^2\right), \\ &\partial_t P = -\gamma P_0 \left(\partial_x v_x + \partial_z v_z\right), \\ &\partial_t \rho = -\rho_0 \left(\partial_x v_x + \partial_z v_z\right). \end{aligned} \tag{2}$$

The adiabatic approximation of the energy equation with the adiabatic exponent γ is used here.

Equations (1) are for Alfvén waves. Suppose that there is an initial disturbance that generates only Alfvén waves. In this case, perturbations of the x and z components of the magnetic field and velocity, as well as plasma pressure and density, can be produced only by the Alfvén wave magnetic pressure, according to (2). Our goal is to describe these perturbations. Note that when deriving Equations (1), (2), no restrictions are required directly on the Alfvén wave amplitude. Nonetheless, since the perturbations described by system (2) are produced by the Alfvén wave magnetic pressure and their amplitudes, hence are characterized by a dimensionless quantity $b_y^2 = \left(B_y / B_0\right)^2$, in order to meet the condition of smallness of these perturbations it is necessary to fulfill the condition $b_y^2 \ll 1$.

2. ALFVÉN WAVES FROM INITIAL PERTURBATION

Suppose that the initial perturbation is a shift of plasma with a velocity $v_y(x, z, 0) = V_0(x, z)$, and there is no field perturbation at the initial moment: $B_y(x, z, 0) = 0$, then for a dimensionless perturbation of the y component of the Alfvén wave magnetic field $b_y = B_y / B_0$ we obtain

$$b_y(x, z, t) = b_{y+}(x, z, t) + b_{y-}(x, z, t),$$

where

$$b_{y+}(x, z, t) = -b(x, z + V_a t),$$

$$b_{y-}(x, z, t) = b(x, z - V_a t),$$

and

$$b(x, z) = -V_0(x, z)/(2V_a),$$

where $V_a = B_0 \big/ \sqrt{4\pi\rho_0}$ is the Alfvén velocity.

The magnetic pressure of such a perturbation is characterized by a value b_y^2, which can be written as

$$b_y^2(x, z, t) = b_{y\pm}^2(x, z, t) + \overline{b_y^2}(x, z, t),$$

where

$$b_{y\pm}^2(x, z, t) = b_{y+}^2(x, z, t) + b_{y-}^2(x, z, t),$$

$$b_{y+}^2(x, z, t) = b^2(x, z + V_a t),$$

$$b_{y-}^2(x, z, t) = b^2(x, z - V_a t),$$

and

$$\begin{aligned} \overline{b_y^2}(x, z, t) &= 2 b_{y+}(x, z, t) b_{y-}(x, z, t) = \\ &= -2b(x, z + V_a t) b(x, z - V_a t). \end{aligned}$$

Thus, the Alfvén wave magnetic pressure is the sum of magnetic pressures in each of the waves propagating in opposite directions and the magnetic pressure resulting from the interaction of these waves. It is clear that $\overline{b_y^2}(x, z, t)$ exists until the plasma perturbation regions in each Alfvén wave intersect. Assume that the region with significant $b(x, z)$ is finite; in this case, the magnetic pressure of the interacting Alfvén waves is finite too.

3. PERTURBATIONS GENERATED BY MAGNETIC PRESSURE OF ALFVÉN WAVES

Perturbations generated by the magnetic pressure of Alfven waves are described by system of equations (2) with zero initial conditions:

$$B_x(x, z, 0) = B_z(x, z, 0) = 0,$$

$$v_x(x, z, 0) = v_z(x, z, 0) = 0,$$

$$\rho(x, z, 0) = 0, \; P(x, z, 0) = 0.$$

Denote

$$L_f = \partial_t\partial_t - V_a^2\Delta,$$
$$L_a = \partial_t\partial_t - V_a^2\partial_z\partial_z, \quad (3)$$
$$L = L_f\partial_t\partial_t - V_s^2 L_a\Delta.$$

In Formulas (3), $\Delta = \partial_x\partial_x + \partial_z\partial_z$, and $V_s = \sqrt{\gamma P_0/\rho_0}$ is the sound velocity.

The equations, derived from (2) in view of (3), which we need later on, are written as

$$Lv_z = -V_a^2 L_f\partial_t\partial_z b_y^2/2 \quad (4)$$

and

$$L\mathrm{div}\mathbf{v} = -V_a^2 L_a\Delta\partial_t b_y^2/2. \quad (5)$$

Equations (2) are for MHD perturbations (with an external force); from them, by virtue of ∂_y=0, Alfvén wave equations (1) were separated. Therefore, to the operator L corresponds the dispersion

$$\omega^4 - \omega^2\left(V_s^2 + V_a^2\right)\mathbf{k}^2 + V_s^2V_a^2k_z^2\mathbf{k}^2$$

with two branches of MHD oscillations remaining after the separation of Alfvén waves — fast magnetic sound

$$\omega^2 = \omega_f^2 = \frac{\mathbf{k}^2\left(V_s^2+V_a^2\right)}{2} + \sqrt{\frac{\mathbf{k}^4\left(V_s^2+V_a^2\right)^2}{4} - V_s^2V_a^2k_z^2\mathbf{k}^2}$$

and slow magnetic sound

$$\omega^2 = \omega_c^2 = \frac{\mathbf{k}^2\left(V_s^2+V_a^2\right)}{2} - \sqrt{\frac{\mathbf{k}^4\left(V_s^2+V_a^2\right)^2}{4} - V_s^2V_a^2k_z^2\mathbf{k}^2}.$$

We designate $\mathbf{k} = (k_x, k_z)$.

We will further consider perturbations whose transverse scale is much smaller than the longitudinal one. In this case, i.e. for $k_x^2 \gg k_z^2$, the dispersion relations for FMS and SMS waves can be represented as

$$\omega_c^2 = k_z^2V_c^2,\ \omega_f^2 = k_x^2V_f^2,$$

where $V_c = V_sV_a/\sqrt{V_s^2+V_a^2}$ is the velocity of SMS waves, $V_f = \sqrt{V_s^2+V_a^2}$ is the velocity of FMS waves.

Rewrite Equation (5), considering that the perturbations for which $\mathrm{div}\mathbf{v} \neq 0$, are generated only by the pressure of interacting Alfvén waves:

$$L\mathrm{div}\mathbf{v} = -V_a^2 L_a\Delta\partial_t\overline{b_y^2}/2. \quad (6)$$

For the perturbations with $\mathrm{div}\mathbf{v} = 0$, use (4) in the form of

$$Lv_z = -V_a^2 L_f\partial_t\partial_z b_{y\pm}^2/2. \quad (7)$$

Thus, Alfvén waves produce perturbations of two types — with $\mathrm{div}\mathbf{v} = 0$ and $\mathrm{div}\mathbf{v} \neq 0$, produced by their magnetic pressure parts $b_{y\pm}^2$ and $\overline{b_y^2}$ respectively.

Analyze the perturbation generated by the non-interacting Alfvén waves. From (7) easily follows the solution $v_{z\pm}(x, z, t)$ satisfying the condition $L_a v_{z\pm} = 0$. For this solution, (7) takes the form

$$\partial_t v_{z\pm} = -V_a^2\partial_z b_{y\pm}^2/2. \quad (8)$$

From (8), we get

$$v_{z\pm} = v_{z+} + v_{z-},\ v_{z+} = -V_a b^2\left(x, z+V_at\right)/2,$$
$$v_{z-} = V_a b^2\left(x, z-V_at\right)/2. \quad (9)$$

As we can see, in each of the waves the direction of the longitudinal flow component coincides with the direction of wave propagation.

As for the transverse flow component, it may be neglected. Since from (6) it follows that in propagating Alfvén waves the condition $\mathrm{div}\mathbf{v} = 0$ holds for the flow, and we analyze perturbations whose transverse scale is much smaller than the longitudinal one, we have $v_x \ll v_z$, so the plasma flow in each of the Alfvén waves can be considered longitudinal. Thus, plasma in an Alfvén wave moves in the direction of its propagation, and its velocity is determined by the ratio of the wave field B_y to the external magnetic field B_0 and by the Alfvén velocity in the wave propagation region. Note that Equation (8) and Formula (9) coincide with the corresponding equalities for the cold plasma flow in propagating Alfvén waves, hence we obtain that the presence of unperturbed plasma pressure does not affect plasma motion in such waves.

It is easy to see that besides the flow the magnetic pressure of Alfvén waves causes the longitudinal magnetic field to change. For a nondimensionalized change of the field $b_{z\pm}$ in propagating Alfvén waves, from second equation (2) and $\mathrm{div}\mathbf{v} = 0$ we obtain $b_{z\pm} = \pm V_a^{-1}v_{z\pm}$, which yields

$$b_{z\pm} = b_{z+} + b_{z-},\ b_{z+} = -b^2\left(x, z+V_at\right)/2,$$
$$b_{z-} = -b^2\left(x, z-V_at\right)/2.$$

Thus, in propagating Alfvén waves the longitudinal magnetic field decreases.

Now consider the perturbation generated by interacting Alfvén waves. Simple transformations of the right-hand side reduce Equation (6) to the form

$$L\mathrm{div}\mathbf{v} = -4V_a^4\partial_t\Delta\left[\left(\partial_z b\left(z-V_at\right)\right)\partial_z b\left(z+V_at\right)\right]. \quad (10)$$

As follows from the above dispersion equation for the L operator, to this operator correspond FMS and SMS waves. Since we are interested in perturbations that can propagate along the magnetic field, we take SMS waves for further consideration. To get them from (10), assume that the perturbation satisfies the condition $\partial_t\partial_t \ll V_a^2\partial_x\partial_x$, which in the case of a transversely small-scale perturbation $\left(\partial_x\partial_x \gg \partial_z\partial_z\right)$ allows us to obtain from (10) an equation for SMS waves of the form of

$$\left[\partial_t\partial_t - V_c^2\partial_z\partial_z\right]\mathrm{div}\mathbf{v} = 4\frac{V_a^4}{V_s^2+V_a^2}\partial_t g,$$
$$g = \left(\partial_z b\left(z - V_a t\right)\right)\partial_z b\left(z + V_a t\right).$$

The solution of this equation can be written as

$$\mathrm{div}\mathbf{v} = \frac{2V_a^2V_c}{V_s^2}\left(\int_z^{z+V_c t} g\left(s, -\frac{s-z}{V_c}+t\right)ds + \right.$$
$$\left. + \int_{z-V_c t}^{z} g\left(s, \frac{s-z}{V_c}+t\right)ds\right). \quad (11)$$

In the case of an initial perturbation having finite spatial dimensions, we can change the limits of integration in (11) for large t as follows:

$$\mathrm{div}\mathbf{v} = \frac{2V_a^2V_c}{V_s^2}\left(\int_{-\infty}^{z+V_c t} g\left(s, -\frac{s-z}{V_c}+t\right)ds + \right.$$
$$\left. + \int_{z-V_c t}^{\infty} g\left(s, \frac{s-z}{V_c}+t\right)ds\right).$$

The expression describes oppositely propagating SMS waves.

The results of calculations for SMS waves are presented in Figures 1 and 2 for different ratios between Alfvén and SMS velocities. To the initial perturbation corresponds the function $b(x, z)$ of the form of $b(x,z) = b_0\exp\left(-z^2/l^2\right)X(x)$. The transverse structure $X(x)$ can be arbitrary. It does not change during wave propagation, so it is not specified and is not reflected in the Figures. The factor b_0 sets the perturbation amplitude; b_0=0.3 was taken for the calculations. The longitudinal velocity in this case has the form

$$v_{z\pm} = -V_{z0}\exp\left(-2\left(z+V_a t\right)^2/l^2\right)X^2(x) +$$
$$+V_{z0}\exp\left(-2\left(z-V_a t\right)^2/l^2\right)X^2(x),$$
$$V_{z0} = V_a b_0^2/2.$$

Figure 1 shows the calculations of div**v** according to (11); and Figure 2, the corresponding calculations of the density perturbation according to the formula

$$\rho/\rho_0 = -\int_0^t \mathrm{div}\mathbf{v}(x, z, \tau)d\tau,$$

which follows from the last equation of (2).

The calculations use the Alfvén velocity and the sound velocity characteristic of different magnetotail regions. In Figures 1, *a* and 2, *a*, the Alfvén velocity is higher than the sound velocity, which corresponds to the parameters of magnetotail lobes and adjacent plasma sheet boundary layers: V_a=6000 km/s, V_a=4000 km/s, V_a=2000 km/s, V_s=1000 km/s. According to the selected values of the Alfvén velocity for V_{z0} characterizing the longitudinal flow velocity, we obtain V_{z0}=270 km/s, V_{z0}=180 km/s, V_{z0}=90 km/s. In Figures 1, *b* and 2, *b*, the Alfvén velocity is equal to or lower than the sound velocity, which corresponds to the parameters of the plasma sheet and the adjacent part of its boundary layer: V_a=1000 km/s, V_a=800 km/s, V_a=600 km/s, V_s=1000 km/s. In this case, for V_{z0} we get smaller values: 45, 36, 27 km/s.

The Figures also indicate that the SMS waves have a sufficiently high amplitude for all the parameters selected. Note that at other parameters being equal, the SMS-wave amplitude is higher at a higher Alfvén velocity. This could argue for the predominance of the effect of excitation of SMS waves together with Alfvén waves in magnetotail lobes, if not for the fact that in these regions the velocity of SMS waves almost coincides with the thermal velocity of ions, which should lead to their fairly rapid absorption. It is therefore more likely that the SMS generation effect considered occurs in the plasma sheet boundary layers. In this region, the Alfvén velocity is quite high, and the SMS-wave and sound velocities differ markedly. Moreover, the high plasma density makes its small relative perturbations noticeable.

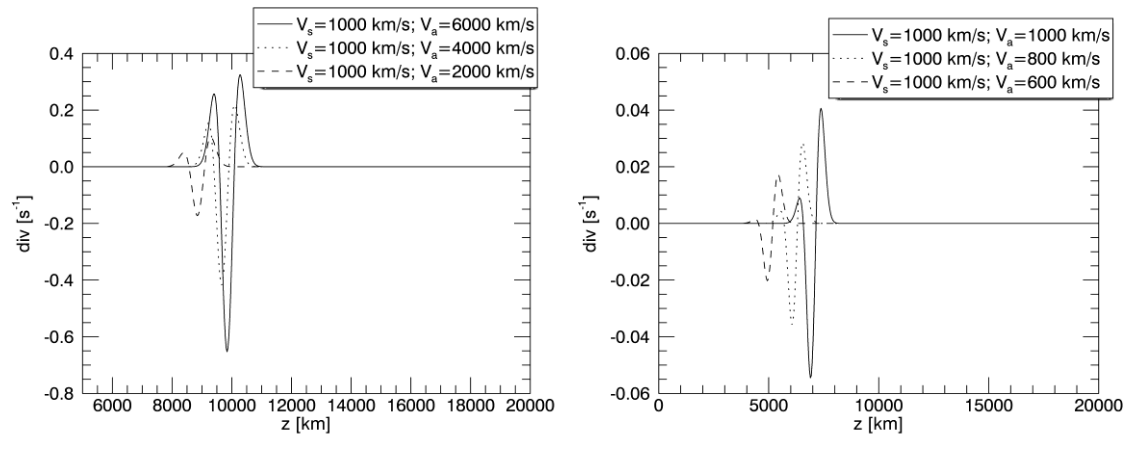

Figure 1. div**v** as a function of the longitudinal coordinate at V_s=1000 km/s, V_a=6000 km/s, V_a=4000 km/s, V_a=2000 km/s, b_0=0.3, t=10 s (*a*); div**v** as a function of the longitudinal coordinate at V_s=1000 km/s, V_a=1000 km/s, V_a=800 km/s, V_a=600 km/s, b_0=0.3, t=10 s (*b*)

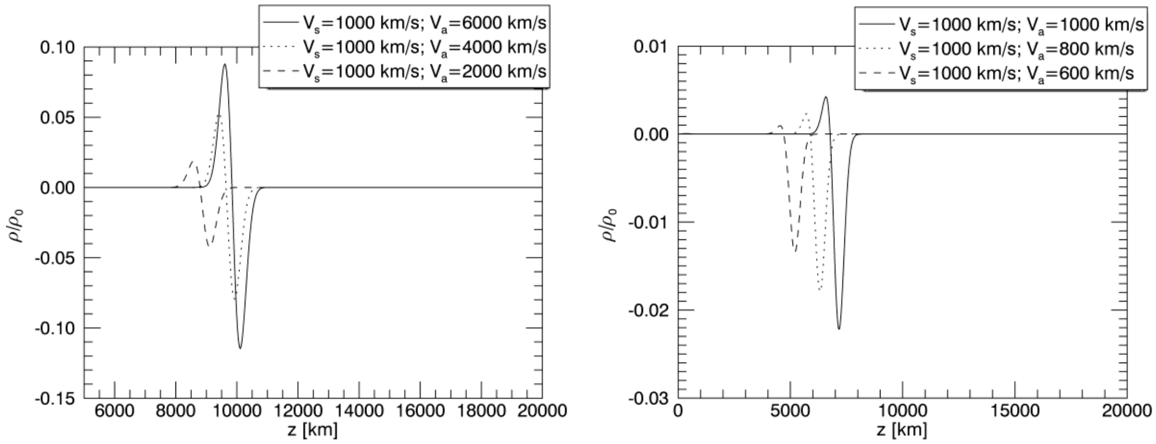

Figure 2. Plasma density perturbation ρ/ρ_0 as a function of the longitudinal coordinate at V_s=1000 km/s, V_a=6000 km/s, V_a=4000 km/s, V_a=2000 km/s, b_0=0.3, t=10 s (*a*); plasma density perturbation ρ/ρ_0 as a function of the longitudinal coordinate at V_s=1000 km/s, V_a=1000 km/s, V_a=800 km/s, V_a=600 km/s, b_0=0.3, t=10 s (*b*)

CONCLUSION

Considering the second-order amplitude perturbations in MHD equations introduce significant changes in the pattern of Alfvén wave propagation. In terms of the linear approximation, the initial transverse plasma shift causes only Alfvén waves, i.e. transverse plasma shifts and transverse magnetic field disturbances propagating along the magnetic field. In the second order in amplitude, for such an initial perturbation we obtain that in oppositely propagating Alfvén waves the magnetic pressure of these waves generates longitudinal plasma flows and causes the external magnetic field to decrease. These results have been received earlier in [Dmitrienko, 2019], using the cold plasma model, and, as shown in this paper, are also valid for finite pressure plasma. The plasma flows generated by Alfvén waves are interesting in terms of interpretation of fast plasma flows observed in the magnetotail. It is significant that fast plasma flows are observed simultaneously with Alfvén waves. In this regard, it is widely believed that fast plasma flows generate Alfvén waves observed along with them. Nonetheless, as we can see from [Dmitrienko, 2019] and this paper, the situation may be reverse: propagating Alfvén waves induce plasma flows in the region of their localization. The differentiation in observations of the plasma flows produced by Alfvén perturbations and the plasma flows generated independently of Alfvén waves should be based on the fact that the region occupied by a plasma flow generated by an Alfvén perturbation moves at the Alfvén velocity, not at the flow velocity. Furthermore, the velocity of the plasma flow produced by Alfvén waves is associated in a certain way with the Alfvén perturbation amplitude. This relationship at the boundary of applicability of the weakly nonlinear approximation gives an estimated value of the order of the Alfvén velocity for the flow velocity. Thus, in the lobes and the adjacent PSBL parts, where the Alfvén velocity is 6000 km/s, flow velocities may be of the order of 1000 km/s. This means that in the approximation used in this work, the longitudinal flow velocity can, broadly speaking, be sufficiently high to cover a range of velocities of fast plasma flows in the magnetosphere such as BBF (bursty bulk flows) and field-aligned beams observed in plasma sheet boundary layers. Note that when considering BBF as an object of application of the results obtained in this work, we should, of course, keep in view their part that is not related to substorms and the global restructuring of the magnetotail. The presence of plasma flows in propagating Alfvén waves is also of interest from the point of view of interpreting proton auroral precipitation.

Consideration of the case of finite pressure plasma, in addition to confirming the presence of second-order effects occurring in cold plasma, also allows us reveal an, effect specific for finite pressure plasma. Such plasma, as is known, has conditions for SMS wave propagation.

That is why the magnetic pressure of Alfvén waves, propagating in opposite directions from their source, at the initial stage of their interacton produces SMS waves that also propagate in opposite directions from the source.

The equations derived for them allow us to establish a link between amplitudes of Alfvén and SMS waves and their spatial structure. The occurrence of SMS waves along with Alfvén waves is of interest not only in itself but also due to the fact that their existence, can, in general, provide information on the location of the Alfvén perturbation source. In the geomagnetic tail, the role of the initial disturbance for Alfvén waves can be played by various rapid deformations in the current sheet region, specifically reconnection processes. These events are of great interest as elements triggering, or at least marking, various processes in the magnetosphere, including global ones (auroral phenomena, restructuring of the current system in the magnetosphere, changing the configuration of the geomagnetic tail). Of undoubted interest is the information on their location in the magnetotail. Generally speaking, such information can be obtained from the difference in the time of recording of Alfvén and SMS waves coming from the same region of sources of these waves, which allows us to determine the distance from the observation point to region of the initial disturbance.

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