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NEW CAPABILITIES OF CHETAEV'S MODEL

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This paper considers anomalies in the magnetotelluric field in the Pc3 range of geomagnetic pulsations. We report experimental data on Pc3 field recordings which show negative (from Earth's surface to air) energy fluxes S_z<0 and reflection coefficients |Q|>1. Using the model of inhomogeneous plane wave (Chetaev's model), we try to analytically interpret anomalies of energy fluxes. We present two three-layer models with both electric and magnetic modes satisfying the condition $|Q^h|>1$. Here we discuss a possibility of explaining observable effects by the resonance interaction between inhomogeneous plane waves and layered media.

Keywords: *Magnetotelluric sounding, geomagnetic pulsations, impedances of layered media, mathematical experiment.*

INTRODUCTION

Experimental studies of a magnetotelluric (MT) field reveal a wide range of effects inconsistent with the model of homogeneous vertically-incident plane wave [Tikhonov, 1950]. First of all, it is a generally observed horizontal inhomogeneity of the MT field that was traditionally attributed to the presence of horizontal inhomogeneities of a medium. In 1969, Prof. Dmitry Nikolaevich Chetaev [Chetaev, 1969] offered a model describing locally the MT field in the vicinity of a point of observation of an inhomogeneous plane wave propagating horizontally along Earth's surface. "The possibility for describing locally a field of just a plane wave," authors [Chetaev et al., 1980] wrote, "stems from its continuity. In the sufficiently small vicinity with linear spatial variations in field components, it is natural to approximate the field by the plane wave determined by these linear increments of complex amplitudes." Subsequently, Chetaev developed a directional analysis. Background and history of the directional analysis (Chetaev's model) are described in detail in [Chetaev et al., 1980; Chetaev, 1985]. Its key idea is to study phase velocities of horizontal propagation of MT field, spatial damping, wave propagation direction (directional angle), elements of polarization ellipses, and apparent resistance of a section by determining complex components of wave vector k_x, k_y from Maxwell equations (in a layer with conductivity σ). The directional analysis gives an instrument for separating the complete MT field into five-component h- (H_{zz}≠0) and e- (E_z≠0) modes. Careful experiments [Chetaev et al., 1980] explained the horizontal inhomogeneity of MT field not by the effect of horizontal inhomogeneities of a medium but only by properties of an inhomogeneous plane wave realized by superposition of horizontally propagating inhomogeneous h- and e-modes. Although the answer to the question about the nature of the E_z component is still ambiguous [Anisimov et al., 1993], specialists already cast no doubt on the possibility of applying the directional analysis to high-resistance sections.

There are at least two effects in the MT field that remain theoretically unjustified. They are an unusual direction (from Earth's surface to air) of the vertical component of Poynting vector S_z and anomalies of the reflection coefficient Q. Such facts were found in a number of experimental studies (see below). The paper examines anomalous energy fluxes S_z<0 and reflection coefficients $|Q|$>1 in terms of the directional analysis.

Table 1. Energy parameters of the MT field of geomagnetic pulsations. Volchinka, July 1988.

Number	Time interval	T, s	ReS_z	ImS_z	$\|Q^h\|$	$\|Q^e\|$
15-	11.14.37–11.15.05	24.7	–44.70	6.70	0.95	1.4
15	15.36.14–15.37.02	23.7	–102.00	49.40	–	–
15	16.29.10–16.30.04	81.0	1.14	–1.86	0.73	4.5
16	0.357.40–04.02.03	95.0	41.90	–38.80	0.59	1.4

Indeed, with a plane electromagnetic wave incident on the boundary of a horizontally stratified medium, an instantaneous vertical energy flux $S_z(t)$=[Re$\mathbf{E}(t)$, Re$\mathbf{H}(t)$]$_z$, which probably has no definite physical meaning, can be both positive and negative [Stratton, 1948]. However, the time average value

$$\bar{S}_z = 1/2\mathrm{Re}[\mathbf{E}, \mathbf{H}^*]_z, \qquad (1)$$

(where * is a sign of complex conjugation) defining the downward direction of energy propagation should remain positive. Conditions

$$\bar{S}_z \geq 0, |Q| = |A_{\text{отр}} / A_{\text{пад}}| \leq 1, \qquad (2)$$

where Q is a reflection coefficient, A_{inc} and A_{ref} are incident and reflection amplitudes, are beyond questions in a 1D model.

The situation when both inequalities (2) are not fulfilled is examined in [Savin, Izrailsky, 1991]. By analyzing results of the mathematical modeling with the use of Chetaev's model, the authors found a three-layer model for which the modulus of electric mode reflection coefficient $|Q^e|$>1. Still, the question about the magnetic mode remained open. Studies results of which are presented bellow show that there is a class of so-called resonance models with both the electric and magnetic modes exceeding 1, i.e. the condition $|Q^h|$>1 is met. This paper reports results of numerical calculations of reflections of magnetic and electric modes in three-layer (resonance) models. We discuss a question of how appropriate it is to interpret directional magnetotelluric sounding (MTS) data by a method of downward analytical continuation of reflection coefficients [Chetaev et al., 1984].

EXPERIMENTAL FACTS

The possibility for obtaining anomalous (from Earth's surface to air) values of S_z<0 was first recognized in [Shaub et al., 1976]. The authors tried to explain the discovered paradox by motion of local magnetic and electric dipoles or by any other instability. Anomalous energy fluxes for a wide range of individual Pc3, Pc4 geomagnetic pulsations [Guglielmi, 1973] were studied by Shaub [Shaub, 1982] using observations made in the village of Tatyanovka (Primorsky Territory). Such a situation was observed during field studies in the village of Volchinka in northern Sakhalin in 1991. Employing formalism of the directional analysis [Chetaev, 1985], the authors of [Savin et al., 1991] discovered not only negative vertical energy fluxes for Pc3 pulsations but also anomalous, exceeding 1, moduli of reflection coefficients for partial waves of electric $|Q^e|$ and magnetic $|Q^h|$ types (Table 1). The experiment in synchronous registration of the MT field at three observation points on the Ukrainian crystalline shield, which was carried out in 1974 under the direction of Chetaev, found the same anomalies. This is confirmed by data given in Table 2 that presents a sample of anomalous moduli of reflection coefficients $|Q^e|$ and $|Q^h|$ for 14 individual Pc3 pulsations numbered as in Table 1 (50 pulsation in total) from [Chetaev et al., 1980]. Calculations were performed for specific resistances (800, 1000, and 1200 ohm·m) of the upper layer of the section.

Table 2. Moduli of Pc3 reflection coefficients as deduced from observations on the Ukrainian crystalline shield.

Pulsation number	R_{01}=800 ohm·m		R_{02}=1000 ohm·m		R_{03}=1200 ohm·m	
	Q^e	Q^h	Q^e	Q^h	Q^e	Q^h
1	0.481	1.020	0.480	1.020	0.470	1.020
5	0.661	2.090	0.661	2.070	0.661	2.060
13	0.557	1.760	0.557	1.670	0.557	1.750
17	0.538	3.390	0.538	3.470	0.539	3.550
18	0.541	3.260	0.540	3.270	0.539	3.270
21	0.418	1.280	0.416	1.280	0.413	1.290
22	0.394	1.050	0.391	1.050	0.413	1.050
40	0.395	1.290	0.392	1.290	0.399	1.290
41	0.396	1.230	0.366	1.230	0.361	1.240
43	0.370	1.070	0.367	1.070	0.364	1.070
45	0.395	1.360	0.392	1.360	0.389	1.370
46	1.040	1.910	1.040	1.910	1.040	1.810
47	0.286	1.290	0.287	1.290	0.288	1.308
50	0.293	1.070	0.291	1.070	0.289	1.070

A question arises whether the facts represent a natural phenomenon or a systematic experimental error. The former assumption is supported by the fact that the MT-field energy anomalies were found by independent researchers under different geoelectric conditions. Nevertheless, we should not reject the second assumption.

Failure of energy conservation law (2) is incompatible with properties of a homogeneous (vertically incident) plane wave; therefore, when interpreting the MT field using the Tikhonov–Cagniard model, we should reject all "inconvenient" geomagnetic pulsations (e.g., those listed in Tables 1, 2). If the experimental error is excluded, we face a situation that requires theoretical explanation.

Is it admissible to automatically apply the energy laws of vertically incident (homogeneous) plane waves to horizontally propagating (inhomogeneous) plane waves? This question is important for practical applications. It suffices to remember that the method of interpreting the MT field by downward analytical continuation of reflection coefficients [Chetaev et al., 1984] relies just on the fulfillment of conditions $\overline{S}_z = 0$ and $|Q^{e,h}(h)|=1$ as limiting and determining depths of occurrence of geoelectric layers at $z=h$.

These conditions are used to reject individual geomagnetic pulsations unsatisfying the said conditions. In view of the results of the study of the magnetic mode [Savin, Izrailsky, 1991], acceptance of the validity of negative energy fluxes for the electric mode and anomalous reflection coefficients $|Q^e|$ limits capabilities of the interpretation method. It is therefore interesting to try to explain the energy anomalies for the magnetic mode, using Chetaev's model.

ENERGY FLUXES IN CHETAEV'S MODEL

Write the equations for real $\mathrm{Re}Z^{e,h}$ and imaginary $\mathrm{Im}Z^{e,h}$ parts of electric and magnetic impedances [Chetaev, 1985] as

$$\mathrm{Re}Z^e=\{R+[R^2+(\omega\mu\sigma-J)^2]^{1/2}\}^{1/2}. \quad (3)$$

$$\mathrm{Im}Z^e= -(\omega\mu\sigma-J)/\{R+[R^2+(\omega\mu\sigma-J)^2]^{1/2}\}^{1/2}. \quad (4)$$

$$\mathrm{Re}Z^h=(\omega\mu\sigma-J)/[R^2+(\omega\mu\sigma-J)^2]^{1/2}\{R+[R^2+(\omega\mu\sigma-J)^2]^{1/2}\}^{1/2} \quad (5)$$

$$\mathrm{Im}Z^h=-\{R+[R^2+(\omega\mu\sigma-J)^2]^{1/2}\}^{1/2}/[R^2+(\omega\mu\sigma-J)^2]^{1/2}. \quad (6)$$

Here $R=\mathrm{Re}(k^2_x+k^2_y)$, $J=\mathrm{Im}(k^2_x+k^2_y)$; $k_x=\alpha_x+i\beta_x$ and $k_y=\alpha_y+i\beta_y$ are horizontal components of the complex wave vector **k**.

Formulas (3)–(6) show that in the vicinity of $J_{\text{кр}}=\omega\mu\sigma$ the partial impedances change abruptly, and the functions $\mathrm{Im}Z^e$ and $\mathrm{Re}Z^h$ reverse sign; if $J>J_{\text{кр}}$,

$$\mathrm{Im}Z^e>0,\ \mathrm{Re}Z^h<0. \quad (7)$$

In a diffusion approximation, the Poynting theorem [Stratton, 1948] takes the form

$$\mathrm{div}\mathbf{S}=-1/2\sigma|\mathbf{E}|^2+1/2i\omega\mu|\mathbf{H}|^2, \quad (8)$$

where the Poynting vector is $\mathbf{S}=1/2[\mathbf{E},\ \mathbf{H}^*]$.

If a conducting region of volume v is bounded by a closed surface Σ, Joule loss $-1/2\int_v \sigma|\mathbf{E}|^2 dv$

should be compensated by an energy flux $-\mathrm{Re}\oint_{\Sigma}\bar{S}_{\mathrm{n}}d\Sigma$, directed into the volume. Therefore, applying the Causs theorem to relation (8)

$$\int_{v}\mathrm{div}S dv = -\oint_{\Sigma}\bar{S}_{\mathrm{n}}d\Sigma = -1/2\int_{v}\sigma\left|\mathbf{E}\right|^{2}dv + 1/2i\omega\mu\int_{v}\left|\mathbf{H}\right|^{2}dv, \tag{9}$$

we get

$$\mathrm{Re}\oint_{\Sigma}\bar{S}_{\mathrm{n}}d\Sigma > 0,\ \mathrm{Im}\oint_{\Sigma}S_{\mathrm{n}}d\Sigma < 0. \tag{10}$$

From (10) it follows that the closed-surface average normal vector component **S**, $<S_{\mathrm{n}}>$ without field sources should satisfy inequalities

$$\mathrm{Re}\oint_{\Sigma} < \bar{S}_{\mathrm{n}} > \geq 0,\ \mathrm{Im} < \bar{S}_{\mathrm{n}} > \leq 0. \tag{11}$$

If a half-space is taken as volume V and the Z axis is downward, we have

$$\mathrm{Re} < \bar{S}_{z} > \geq 0,\ \mathrm{Im} < \bar{S}_{z} > \leq 0. \tag{12}$$

Clearly, the first of the inequalities is in accord with notions about downward energy distribution in the direction of a wave incident to z=0.

As indicated above, the directional analysis is based on separation of the total MT field into magnetic and electric partial waves (modes). Hence, it is natural to examine the energy fluxes S_z^{h} and S_z^{e} individually for each of the h- and e-modes. Heeding the equations for the partial impedances $Z^{\mathrm{e,\,h}}$ (see (21), [Chetaev, 1985]), we derive $E_x^{\mathrm{e,\,h}}=Z^{\mathrm{e,\,h}}H_y^{\mathrm{e,\,h}}$, $E_y^{\mathrm{e,\,h}}=-Z^{\mathrm{e,\,h}}H_x^{\mathrm{e,\,h}}$.

Given Formula (1), for partial waves of e and h types we have

$$S_z^{\mathrm{e,\,h}}=1/2Z^{\mathrm{e,\,h}}(|H_x^{\mathrm{e,\,h}}|^2+|H_y^{\mathrm{e,\,h}}|^2). \tag{13}$$

Thus, signs of real and imaginary parts of the vertical component of the complex Poynting vector $S_z^{\mathrm{e,h}}$ for both the types of partial waves should coincide with signs of the respective parts of partial impedances $Z^{\mathrm{e,\,h}}$.

Founding on (12), we conclude that for any J the following inequalities should hold

$$\mathrm{Re}Z^{\mathrm{e,\,h}}\geq 0,\ \mathrm{Im}Z^{\mathrm{e,\,h}}\leq 0. \tag{14}$$

These inequalities correspond to functions (3)–(6) throughout the domain of definition of J except for $\mathrm{Im}Z^{\mathrm{e}}$ and $\mathrm{Re}Z^{\mathrm{h}}$ such that at $J>J_{\text{кр}}$ they should satisfy conditions (7). Consequently, for $J>J_{\text{кр}}$

$$\mathrm{Re}\,\bar{S}_{\mathrm{n}}^{\mathrm{h}} < 0,\ \mathrm{Im}\,S_{\mathrm{n}}^{\mathrm{e}} > 0, \tag{15}$$

i.e. both inequalities (12) are violated.

Thus, with an inhomogeneous plane wave propagating in a layered conducting medium, we face a paradoxical situation such that at $J>J_{\text{кр}}$ the partial magnetic wave energy flows upward $\mathrm{Re}\,\bar{S}_z^{h} < 0$, contrary to intuitive notions. In addition, $\mathrm{Im}\,\bar{S}_z^{e} > 0$, i.e. the second of inequalities (11) is violated.

Our analysis of the experimentally found anomalous energy fluxes of Pc3 geomagnetic pulsations allows us to theoretically explain at least some of the data.

Furthermore, let us try to answer a question of how correctly conditions (12) were derived.

To do this, we used Causs theorem (9) applied to infinite volume (half-space). For an inhomogeneous plane wave at kh, $y \to -\infty$ the integrand tends to infinity because

$$\mathbf{E}, \mathbf{H} \sim \exp\left[i\left(k_x x + k_y y\right)\right] \sim \exp\left[-\left(\beta_x x + \beta_y y\right)\right] \to \infty, \beta_{x,\,y} > 0,$$

and their respective integrals diverge. Hence, it is incorrect to apply the Gauss theorem in the model of interest; and it seems that the «natural» downward direction of the energy flux $\overline{S}_z > 0$ does not follow from the Poynting theorem.

It is therefore reasonable (and is likely to be the only correct way) to rely only on the conclusions drawn from inner laws of Chetaev's model, specifically on properties of partial impedances of layered media resulting in conditions (15).

But the anomalous energy fluxes themselves seem to defy common sense. Indeed, the MT field falls from the top downward because its sources are generally thought to be at the top, i.e. in Earth's magnetosphere and ionosphere; and the direction of the energy flux of this filed is directly opposite. Let us try to provide at least some insight into the problem under study.

The questions concerning the electromagnetic field energy flux are known to be perhaps the most difficult in classical electrodynamics. If the Poynting theorem expressing the energy conservation law as a complete integral [Stratton, 1948] is unquestionably valid, the Poynting vector **S** is determined up to an arbitrary solenoidal vector **a**, i.e. ambiguously. "The vector **S is** only a possible expression for energy flux," authors of [Feynman et al., 1977] wrote, "...We have to admit that we are still unaware of how energy is distributed in the electromagnetic field... It is evident that everyday intuition deceives us." We believe that any theoretical research into electromagnetic field energy fluxes will have "original sin" – energy flux uncertainty. It can therefore lead to a discussion that has no prospect of being ever concluded.

NUMERICAL COMPUTATION OF THE REFLECTION COEFFICIENTS

Let us examine the behavior of reflection coefficients in Chetaev's model.

It is convenient to bring the equations that determine the dependence of partial potentials $U^{e,\,h}$ on depth z [Savin, Izrailsky, 1986] into the unified form

$$(zU'/\eta)' - \eta Z_0 U = 0, \qquad (16)$$

where $\eta=\sqrt{k_x^2+k_y^2-i\omega\mu\sigma}$, and Z_0 is the partial specific impedance. Input partial impedances Z for e- and h-type waves can equally be expressed through the partial potential U

$$Z=-Z_0 U'/(U\eta). \tag{17}$$

Inside each layer, $\sigma=const$, therefore the solution of (16) takes the form

$$U(z)=Ae^{-\eta z}+Be^{\eta z}, \tag{18}$$

where A, B are amplitudes of incident and reflected waves.

Introduce [Chetaev et al., 1984] a reflection coefficient $Q(z)$ at z representing a complex ratio of the upward-wave amplitude to the downward-wave amplitude,

$$Q(z)=B/A\exp(2\eta z). \tag{19}$$

Using (17), obtain the following relation between the reflection coefficient $Q(z)$ and the partial impedances Z and Z_0, $Z/Z_0=[1-Q(z)]/[1+Q(z)]$, then

$$Q(z)=(1-Z/Z_0)/(1+Z/Z_0). \tag{20}$$

Since the input (complete) impedance Z continuously depends on z, and the specific impedance Z_0 has discontinuities on the boundary of the layers, $Q(z)$ is a piecewise continuous function.

Formulas (19)–(20) along with the recurrence relations [Dmitriev, 1970] for partial impedances of a layered medium allow us to compute the reflection coefficient at any point of the medium.

In the case of a homogeneous plane wave ($\beta_x=0$, $\beta_y=0$), the reflection coefficient meets the inequality

$$|Q(z)|<1. \tag{21}$$

Let us see whether this inequality holds for an inhomogeneous plane wave.

For a two-layer medium, it is easy to analytically study the inequality

$$\left|Q^{\mathrm{i}}\right|=\left|\left(Z^{\mathrm{i}}_{(0)1}-Z^{\mathrm{i}}_{(0)2}\right)/\left(Z^{\mathrm{i}}_{(0)1}+Z^{\mathrm{i}}_{(0)2}\right)\right|\leq 1, \tag{22}$$

where $Z_{(0)1}$ and $Z_{(0)2}$ are specific partial impedances in the first and second media, i=e, h. Omitting the cumbersome calculations, we focus on the result. At R>0, the modulus of the reflection coefficient of both the modes is always ≤ 1. Still, at R<0 both moduli of the reflection coefficients can be greater than unity.

MATHEMATICAL EXPERIMENT

Furthermore, let us consider geoelectric models for which we made numerical calculations in dimensionless variables. As could be expected, the energy conservation law $|Q^{e,\,h}|<1$ held in most cases. Nevertheless, Savin and Izrailsky [Savin, Izrailsky, 1991] examined a model for which the opposite inequality $|Q^{h}|>1$ holds. However, the question concerning the magnetic mode has remained open so far.

Let us analyze three-layer model (1) that in a first approximation represents the geoelectric section in the village of Volchinka [Nikiforov et al., 1983]: σ_2/σ_1=0.1, σ_3/σ_1=1, h_2/h_1=1. The calculations were made for $r=R/\omega\mu\sigma=10^{-3}$; the dimensionless skin layer $\delta=(2/\omega\mu\sigma)^{1/2}/h_1=10$, $i=J/\omega\mu\sigma$ varied from 0.5 to 1.2; the frequency ω=0.1 Hz.

The behavior of $|Q^h|$ was examined as a function of *i*. Figure 1 plots this function calculated at the bottom of the first layer for $z=h-0$. It is obvious that in the small vicinity of i=1, the reflection coefficient of the magnetic mode has a sharp peak exceeding 1, $|Q^h(i=1)|=1.14$.

We consider another three-layer model (2) with parameters σ_2/σ_1=10, h_2/h_1=1, σ_3/σ_1=1, δ=10, ω=0.1 such that functions $|Q^e|$ (Figure 2) and $|Q^h|$ (Figure 3), calculated at the bottom of the first layer, are also greater than 1, $|Q^e(i=1.01)|=1.42$, $|Q^h(i=1.96)|=1.28$.

The analysis of the plots in view of the results of calculations of the electric mode in model (1) [Savin, Izrailsky, 1991] shows that in the reflection from the conductor–dielectric surface $|Q^e|$ has a wide peak; $|Q^h|$, a narrow one. For reflection from the dielectric–conductor surface, the situation is directly opposite.

The situation with ReS_z<0 can be explained by (13) and well-known properties of partial impedances of inhomogeneous plane waves.

RESULTS

We cannot assert that the reflection coefficient is in an out-of-limit region and vertical energy fluxes become negative for arbitrary layer structures – certainly it is not so. Specificity of the three-layer geoelectric models we considered, which are applied to Sakhalin [Nikiforov, 1985], Kamchatka [Moroz, 1991], and other regions, is that the first and second layers have a roughly equal thickness; the first and third layers, equal conductivity; and conductivity of the second layer is approximately by an order of magnitude lower (model 1)/higher (model 2) than that of the first layer. Such a class of layer structures could be called resonance models.

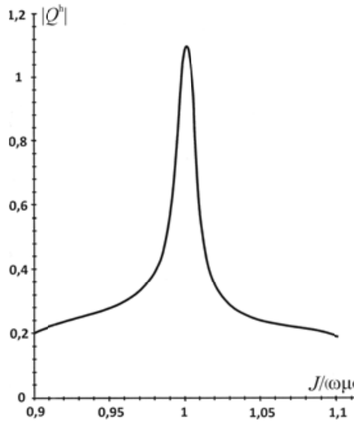

Figure 1. Function $|Q^h|$ for model 1

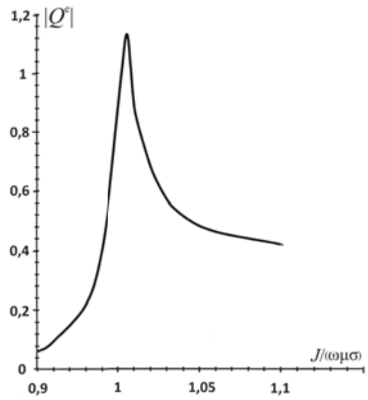

Figure 2. Function $|Q^e|$ for model 2

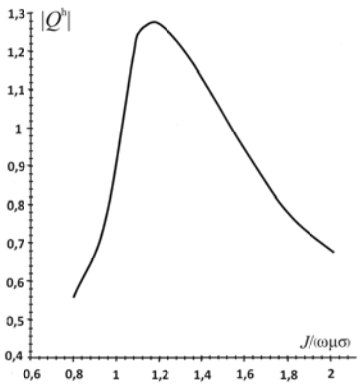

Figure 3. Function $|Q^h|$ for model 2

Let us try to explain the results both in terms of the infinite energy of an inhomogeneous plane wave and in terms of the wave–layer system resonance.

If we assume that the wave energy is finite, from $|Q|>1$ on the entire surface $z=0$ it follows that the reflected wave energy W_{ref} is higher than the incident wave one W_{inc} because $W_{\mathrm{ref}}=|Q|^2 W_{\mathrm{inc}}$. This contradicts the energy conservation law. However, since the inhomogeneous plane wave energy is equal to infinity on one of the semiaxes, e.g. as $x\to-\infty$ and $\beta_x>0$ $A\sim\exp[i(\alpha_x+i\beta_x)x]\to\infty$, $W\sim|A|^2$, there is no contradiction because we compare infinite values.

If we suppose that the MT field varies only in a finite area of the earth's crust, as is the case for the Chetaev wave, the condition $W_{\mathrm{inc}}>W_{\mathrm{ref}}$ does not mean that for this area the energy conservation law does not work. In actual fact, if we bound the vicinity of the observation point (volume V) by a closed surface Σ, the energy inflow into the surface can come not only from the top but from the bottom and from a side as well (see above citations from [Feynman et all, 1977]), i.e. from fairly distant regions outside the area of interest with sufficiently high plane-wave energy. Then we should assume that the Poynting vector **S** changes its direction, thus providing an energy inflow into V.

Let us now turn to the resonance interpretation. At $|Q^{e,\,h}|>1$, we are likely to deal with the domain of geoelectric layer parameters close to values corresponding to eigenfrequencies of the layer system. As the resonance is approached, Q should increase infinitely.

It is therefore interesting to analyze a situation arising from reflection of an acoustic wave from an elastic plate situated in a fluid [Brekhovskikh, 1957]. We examine free waves in the plate, i.e. such waves that can exist with zero (without external excitation) incident-wave amplitude. "If the incident-wave amplitude tends to zero," L.M. Brekhovskikh wrote, "reflected and transmitted wave amplitudes remain finite, and the reflection coefficient can take any prescribed values." There is an analogy between free waves in the plate and surface waves (Rayleigh waves) on the boundary of a rigid body and fluid. "In this case, a wave process takes place that extends along the boundary without an incident wave, i.e. we have a case of a surface wave" (L.M. Brekhovskikh).

In our setting, we also consider a surface electromagnetic, rather than acoustic, wave with complex spatial frequencies. Yet parameters of discovered three-layer model 1 ($\rho_2=10\rho_{1(3)}$) representing anomalous field characteristics can be referred to a plate such that its elastic properties differ from properties of the fluid above and below it. Figure 1 shows that there is a narrow peak of Q^{h} such that it can be attributed to a resonance peak, whereas the peak of Q^{e} is much more flat. In this case, i.e. in reflection from the surface of the conductor–dielectric section, the magnetic mode comes into resonance with model 1. The narrow peak of Q^{e} (Figure 2) in reflection from the dielectric–conductor boundary suggests that the electric mode comes into resonance with model 2.

To explain the results in more detail, recall [Chetaev, 1985] that processes of propagation of electromagnetic fields in layered media exhibit a close analogy to wave propagation in one-dimensional transmission lines comprised of two conductors with oppositely directed currents. Yet to the E mode corresponds a series impedance; to H mode, a shunt admittance. A reader can readily figure out the situation with resonance in both the cases on his/her own because it is unreasonable to consider this issue in this paper owing to the cumbersome calculations.

CONCLUSION

This paper was aimed at reviving interest to Chetaev's model distinguished for its striking simplicity and elegance. Unfortunately, in recent years it has been undeservedly forgotten. We tried to unveil new possibilities of this model for studying the so-called energy MT-field anomalies unexplained by the traditional Tikhonov–Cagniard model. The mathematical experiment we conducted allowed us to explain the apparent violation of the law of energy conservation for the natural geoelectromagnetic field in the Pc3 range of geomagnetic pulsations only by structural properties of electric and magnetic modes with their superposition corresponding to the primary (total) MT field. The results we obtained from the directional analysis raise the possibility to interpret experimentally observed MT fields of a wider class than that provided by the traditional approach. Properties of the natural electromagnetic field, which do not fit into the Procrustean bed of the homogeneous plane wave model but legitimized in the directional MTS model, include a horizontally inhomogeneous field observed virtually everywhere, an exponential variations in the amplitude of Pi2 pulsations [Savin, 1986], and Pc3 geomagnetic pulsations revealing energy anomalies. In the interpretation using the Tikhonov–Cagniard model, all MT field variations with the said properties are rejected. We believe that it would be an inexcusable error for geoelectric researchers to ignore the enormous capabilities of Chetaev's model that enriches traditional interpretation.

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