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NUMERICAL ANALYSIS OF THE SPATIAL STRUCTURE OF ALFVÉN WAVES IN A FINITE PRESSURE PLASMA IN A DIPOLE MAGNETOSPHERE

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Abstract. We have carried out a numerical analysis of the spatial structure of Alfvén waves in a finite pressure inhomogeneous plasma in a dipole model of the magnetosphere. We have considered three magnetosphere models differing in maximum plasma pressure and pressure gradient. The problem of wave eigenfrequencies was addressed. We have established that the poloidal frequency can be either greater or less than the toroidal frequency, depending on plasma pressure and its gradient. The problem of radial wave vector component eigenvalues was considered. We have found points of Alfvén wave reflection in various magnetosphere models. The wave propagation region in the cold plasma model is shown to be significantly narrower than that in models with finite plasma pressure. We have investigated the structure of the main Alfvén wave harmonic when its polarization changes in three magnetosphere models. A numerical study into the effect of plasma pressure on the structure of behavior of all Alfvén wave electric and magnetic field components has been carried out. We have established that for certain parameters of the magnetosphere model the magnetic field can have three nodes, whereas in the cold plasma model there is only one. Moreover, the longitudinal magnetic field component changes sign twice along the magnetic field line.

Keywords: MHD waves, dipole model of the magnetosphere, MHD resonances.

INTRODUCTION

Alfvén waves are very common in Earth's magnetosphere. They are identified with a significant part of ultra-low frequency (ULF) waves in the magnetosphere [Clausen, Yeoman, 2009], play an important role in accelerating charged particles [Mann et al., 2012; Zong et al., 2017; Potapov et al., 2012; Klimushkin et al., 2021], in magnetosphere-ionosphere coupling [Tamao, 1984; Lysak, Song, 2006] and auroral phenomena [Fedorov et al., 2001; Pilipenko et al., 2004; Kostarev et al., 2021; Keiling, 2021]. An important parameter determining the structure of Alfvén waves is the azimuthal wave number *m* [Chen, Hasegawa, 1991; Leonovich, Mazur, 1993]. When it is small (m~1), the phenomenon of resonant excitation of Alfvén waves occurs as follows. A fast magnetic sound (FMS) is excited at the boundary of the magnetosphere by processes in the solar wind [Mazur, Chuiko, 2011; Mishin et al., 2013; Leonovich et al., 2021] and propagates deep into the magnetosphere. Near a magnetic shell, FMS is reflected inward, the superposition of incident and reflected FMS waves forms a standing mode, also known as the global MHD mode. However, part of the FMS energy penetrates into the non-propagation region, exciting there an Alfvén wave on a resonant magnetic shell [Leonovich, Mazur, 2016]. Such a resonant Alfvén wave has a number of observable properties: a sharp peak in amplitude, a phase shift by 180° when passing through a resonant feature, as well as a region with opposite phase delays on the side of the wave source [Glassmeier et al., 1999; Pilipenko et al., 2016]. These conclusions of the theory are confirmed by both ground-based and satellite experiments [Samson, 1988; Agapitov et al., 2009; Pilipenko et al., 2016]. Nonetheless, at large values of the azimuthal wavenumber ($m \gg 1$, azimuthally small-scale waves), the Alfvén wave cannot be excited by resonance with FMS since only an exponentially small part of FMS energy penetrates into the magnetosphere [Guglielmi, Potapov, 1984]. Alfvén waves $m \gg 1$ can therefore be generated only by intramagnetospheric sources such as various plasma instabilities [Karpman et al., 1977; Southwood, 1983] or alternating currents, caused by the drift of substorm clouds of charged particles [Guglielmi, Zolotukhina, 1980; Mager, Klimushkin, 2007]. In this work, we investigate the azimuthally small-scale Alfvén waves.

In terms of wave polarization, there are two extreme cases of Alfvén waves in the magnetosphere, called toroidal and poloidal modes. These modes are characterized by field line fluctuations in azimuthal and radial directions respectively. In accordance with the polarization properties of Alfvén waves, the electric field vector oscillates in the radial and azimuthal directions respectively. In toroidal modes, the radial wavelength is much smaller than the azimuthal one; in poloidal modes, vice versa. Poloidal Alfvén waves can have only large azimuthal wavenumbers ($m \gg 1$), while toroidal ones have both small (m~1) and large ones [Leonovich, Mazur, 1993; Leonovich, Mazur, 2016]. Let us emphasize that the large azimuthal wave number is only a necessary but insufficient condition for the poloidal polarization of Alfvén waves. Even at $m \gg 1$ a wave may have toroidal polarization if the radial wavelength is much smaller than the azimuthal one. This issue has been examined in depth in [Leonovich, Mazur, 1993]. The authors have shown that a monochromatic azimuthally small-scale Alfvén wave that initially has poloidal polarization generally propagates across magnetic shells due to specific dispersion caused by field line curvature. In this case, the azimuthal wavelength remains approximately constant, yet the radial wavelength gradually decreases and eventually turns out to be shorter than the azimuthal one. Accordingly, the wave polarization changes from poloidal to toroidal. Moreover, if an azimuthally small-scale Alfvén wave is generated by a pulsed source, it turns into a toroidal one due to phase dispersion [Mann, Wright, 1995; Leonovich, Mazur, 1998]. Thus, toroidally polarized waves can have both small and large azimuthal wave numbers.

The Alfvén wave structure is often studied using the Wentzel—Kramers—Brillouin (WKB) radial coordinate approximation [Leonovich, Mazur, 1993]. In toroidal modes, the radial wave vector component goes to infinity; in poloidal modes, to zero.

Eigenfrequencies of oscillations in toroidal and poloidal modes differ somewhat [Radoski, 1967]. This phenomenon, also known as polarization spectrum splitting [Guglielmi, 1970], is caused by inhomogeneous curvature of field lines [Krylov, Lifshitz, 1984; Leonovich, Mazur, 1990]. The field line curvature has been found to lead to slow propagation of Alfvén waves across magnetic shells, accompanied by the change of their polarization from poloidal to toroidal [Leonovich, Mazur, 1993; Leonovich et al., 2015]. The influence of azimuthal asymmetry effects on the Alfvén wave structure has been studied in [Klimushkin et al., 1995; Mager, Klimushkin, 2021; Elsden, Wright, 2022; Wright et al., 2022].

Alfvén waves are often observed in regions of the magnetosphere with a considerable amount of hot plasma: the ratio of plasma pressure to magnetic one β can be 0.5 and higher [Mager, 2021]. In a finite pressure plasma, the field line curvature causes Alfvén waves to connect with the slow magnetic sound (SMS) [Southwood, Saunders, 1985; Walker, 1987; Mazur et al., 2014]. Yet, the connection with SMS itself has little effect on Alfvén wave propagation since the characteristic frequencies of SMS are much lower than those of the Alfvén mode [Agapitov et al., 2008]. It is much more important that the finite plasma pressure combined with the magnetic field inhomogeneity leads to a change in the dispersion relation of Alfvén waves, especially in the case of poloidal polarization [Safargaleev, Maltsev, 1986]. The differential equation describing the structure of Alfvén waves in an inhomogeneous plasma has been derived in [Klimushkin et al., 2004].

Another consequence of the finite plasma pressure is the possibility of development of ballooning and flute (permutational) instabilities in magnetospheric plasma [Hameiri et al., 1991; Xing, Wolf, 2007; Cheremnykh, Parnowski, 2006; Mazur et al., 2012; Xia et al., 2017; Rubtsov et al., 2020]. In this paper, we consider the plasma stable with respect to oscillations of this type.

The transverse structure of Alfvén waves at arbitrary azimuthal wave number *m* has been examined in [Klimushkin et al., 2004]. Also of great interest is the longitudinal structure of waves with different values of *m* and different wave polarization. Specifically, this is due to the fact that Alfvén waves play an important role in accelerating high-energy particles of the magnetosphere — particles of radiation belts and ring current. When moving in the geomagnetic field, a particle oscillates along a field line (drift-bounce); therefore, calculating the interaction of waves and particles requires us to know the longitudinal wave structure.

This paper examines the structure of Alfvén waves in a finite pressure plasma with arbitrary polarization. The emphasis is on the fundamental harmonic of the wave standing between ionospheres of conjugate hemispheres since for such waves the polarization splitting of the spectrum is most pronounced and they are regularly observed in experiments [Dai et al., 2013; Mager et al., 2018; Takahashi et al., 2018a, b].

Sections 1 and 2 present the main relations determining the plasma equilibrium and the spatial structure of Alfvén waves. Section 3 delves into three magnetosphere models with different parameters of plasma pressure distribution across magnetic shells. Section 4 discusses toroidal and poloidal eigenfrequencies for these three models. Section 5 addresses the inverse problem: the radial wave vector component for a given wave frequency is found in the transverse WKB approximation. In addition, we numerically analyze the longitudinal structure of the main harmonic and electric and magnetic field components for the three magnetosphere models as a function of the radial wave vector component. Section 6 presents the main results of the work.

1. EQUILIBRIUM

We deal with a two-dimensional inhomogeneous model of the magnetosphere, where plasma is considered inhomogeneous both along magnetic field lines and across magnetic surfaces. First, let us introduce a curvilinear coordinate system $\{x^1, x^2, x^3\}$, where the x^3 coordinate marks the position of a point on a field line, and the other two, x^1 and x^2, are radial and azimuthal coordinates (to represent them, we use the McIlwain parameter $x = L$ and the azimuthal angle $x^2=\phi$). The element of length along the *i*-th coordinate axis is expressed in terms of the increment of the coordinate x^i:

$$dl_i = \sqrt{g_i}\, dx^i,\ i = 1, 2, 3,$$

where $g_i\,(x^1, x^3)$ are the diagonal components of metric tensor (the nondiagonal components are zero due to the orthogonality of the coordinate system). Determinant of the metric tensor is $g=g_1 g_2 g_3$.

The equilibrium magnetic field *B*, the plasma pressure *P*, and the current *J* are related by the hydromagnetic equilibrium condition:

$$J = \frac{4\pi}{B\sqrt{g_1}}\frac{\partial P}{\partial L}. \quad (1)$$

The paper explores the magnetospheric regions where the ratio of plasma pressure to magnetic one $\beta=8\pi P/B^2<<1$. Since the magnetic field strength increases strongly from the equator to the ionosphere, average β along the field line appears to be small even if at the equator this value is only slightly less than unity: $\beta=8\pi P/B^2<1$. Under these conditions, the magnetic field may be considered approximately dipole. Indeed, numerical calculations of the equilibrium have revealed that in the inner magnetosphere (at L=5÷8) the finite pressure effects cause the magnetic field to deviate from the dipole only slightly [Xia et al., 2017].

In the spherical coordinate system r, the θ metric tensor components g_1 and g_2 are written as follows:

$$g_1 = \frac{\cos^2\theta}{1+3\sin^2\theta}, \quad g_2 = L^2\cos^6\theta. \quad (2)$$

The third metric tensor component g_3 is expressed in terms of the length element along the field line:

$$dl_\parallel = \sqrt{g_3}dx^3 = L\cos\theta\sqrt{1+3\sin^2\theta}d\theta.$$

2. BASIC EQUATIONS DESCRIBING ALFVÉN WAVES IN THE DIPOLE MODEL OF THE MAGNETOSPHERE

In an inhomogeneous plasma, three modes of MHD oscillations (Alfvén mode, fast (FMS) and slow (SMS) magnetic sounds) are interconnected. We, however, deal with azimuthally small-scale waves ($m \gg 1$) when characteristic frequencies of FMS (at a given quasiclassical wave vector) are much higher than Alfvén ones. On the other hand, in plasma with $\beta \ll 1$, SMS frequencies, on the contrary, are much lower than Alfvén ones. Under these conditions, the Alfvén mode may be considered separately from FMS and SMS. The electric field of an Alfvén wave is expressed as [Tamao, 1984; Klimushkin, 1994]

$$\vec{E} = -\nabla_\perp\Phi, \quad (3)$$

where $\nabla_\perp$ is the nabla projection on the direction across the magnetic field; Φ is the scalar function, which we call the potential.

The magnetosphere model is assumed to have an azimuthal symmetry, so the potential depends on time and azimuth as exp $i(k_2 x^2-\omega t)$, where $k_2=m$ is an azimuthal wave number, $x^2=\phi$ is an azimuthal angle, ω is a wave frequency. Then, in the WKB radial coordinate approximation, the wave structure is described by the expression

$$\Phi \propto \tilde{\Phi}(l_\parallel)\exp i\int k_1(x^1)dx'^1,$$

where k_1 is the radial wave vector component, which is a function of the radial coordinate [Leonovich, Mazur, 1993]. Note that the WKB approximation implies that the amplitude $\tilde{\Phi}(x^1,x^3)$ depends on the radial coordinate much weaker than the radial component of the wave vector.

In the WKB approximation, the E_a and radial E_r components of the Alfvén wave electric field, measured in the local Euclidean basis, are written as

$$E_a = -i\frac{k_1}{\sqrt{g_1}}\Phi, \quad E_r = -i\frac{k_2}{\sqrt{g_2}}\Phi. \quad (4)$$

Accordingly, the behavior of azimuthal and radial physical components of the magnetic field can be found from the following relations:

$$B_a = k_2\frac{c}{\omega}\frac{1}{\sqrt{g_2}}\frac{\partial\Phi}{\partial l_\parallel}, \quad B_r = -k_1\frac{c}{\omega}\frac{1}{\sqrt{g_1}}\frac{\partial\Phi}{\partial l_\parallel}. \quad (5)$$

In addition, when taking into account the final pressure, the longitudinal magnetic field component proves to be linked to the Alfvén wave [Klimushkin et al., 2004]

$$B_\parallel = \frac{ck_2}{\omega}\frac{1}{\sqrt{g_1 g_2}}\frac{\eta}{2K}\Phi, \quad (6)$$

where

$$\eta = -2K\left(\frac{4\pi}{B_2\sqrt{g_1}}\frac{\partial P}{\partial L} + K\gamma\beta\right), \quad (7)$$

K is the local field line curvature

$$K = \frac{3}{L\cos\theta}\frac{1+\sin^2\theta}{\left(1+3\sin^2\theta\right)^{3/2}}. \quad (8)$$

The wave field of Alfvén modes can be described using the equation derived in [Klimushkin et al., 2004] from a system of linearized MHD equations:

$$\kappa^2\hat{L}_T(\omega)\Phi + \hat{L}_P(\omega)\Phi = 0, \quad (9)$$

where $\kappa^2 = k_1^2/k_2^2$, $\hat{L}_T(\omega)$ and $\hat{L}_P(\omega)$ are toroidal and poloidal differential operators:

$$\hat{L}_T(\omega) = \frac{\partial}{\partial l_\parallel}\sqrt{\frac{g_2}{g_1}}\frac{\partial}{\partial l_\parallel} + \sqrt{\frac{g_2}{g_1}}\frac{\omega^2}{A^2},$$
$$\hat{L}_P(\omega) = \frac{\partial}{\partial l_\parallel}\sqrt{\frac{g_1}{g_2}}\frac{\partial}{\partial l_\parallel} + \sqrt{\frac{g_1}{g_2}}\left(\frac{\omega^2}{A^2}+\eta\right). \quad (10)$$

These operators define the longitudinal structure of toroidal and poloidal Alfvén modes. Note that the poloidal operator $\hat{L}_P(\omega)$ has a component η, associated with plasma pressure and its gradient (7). Due to the high conductivity of ionospheric plasma, an Alfvén wave is reflected from the ionosphere, so the boundary condition for Equation (9) has the form

$$\Phi\left(x_\pm^3\right) = 0. \quad (11)$$

3. NUMERICAL MODEL

For the numerical calculation of the electric potential Φ, use the following pressure profile [Klimushkin et al., 2004]:

$$P = P_0\left[1-\tanh^2\left(\frac{L_0-L}{D}\right)\right], \quad (12)$$

where P_0 is the maximum plasma pressure that is achieved on a magnetic shell L_0=4; D characterizes the pressure profile width. The value P_0 is given by the ratio of plasma pressure to magnetic one $B_{00}^2/(8\pi)$ (B_{00}=0.44 G) at the equator on the magnetic shell L_0 representing the maximum plasma pressure. The plasma density is supposed to monotonically decrease with distance from Earth in accordance with the power law:

$$\rho = \rho_0 \left(\frac{r_0}{r} \right)^3 . \tag{13}$$

We ignore the presence of the plasmapause here.

We consider the values D and β_0 to be two main parameters of magnetosphere models. In models 1 and 2, a change in the plasma pressure P across magnetic shells is relatively sharp, which corresponds to a small value of the parameter D (D=0.5 for model 1 and D=0.7 for model 2). On the contrary, in model 3 the pressure changes relatively smoothly (D=2). The equilibrium current J in the models is localized much more strongly than in model 3. In models 1 and 3, the parameter β_0 is assumed to be the same, β_0=0.105. In model 2, this parameter is taken to be β_0=0.15. The distribution of the equatorial values of P, β, and J across magnetic shells is shown in Figure 1, *a*–*c*. Figure 1, *d* illustrates the distribution of η, included in the definition of poloidal operator (10).

4. TOROIDAL AND POLOIDAL FREQUENCIES AND EIGENFUNCTIONS

To begin with, consider problem (9) as an eigenproblem with respect to the wave frequency ω at a fixed value of κ^2. Examine two extreme cases: $\kappa^2 \to \infty$ and $\kappa^2 \to 0$. In the former case, the Alfvén wave is called toroidal ($E_r \gg E_a$, $B_r \ll B_a$); in the latter, poloidal ($E_r \ll E_a$, $B_r \gg B_a$).

In the former case, Equation (9) is as follows

$$\hat{L}_T(\omega)\Phi = 0 \tag{14}$$

with boundary condition (11). Call the eigenfrequencies Ω_{TN} toroidal (here N is a longitudinal wave number). The toroidal eigenfrequencies depend on the radial coordinate x^1 as on a parameter. If the wave frequency ω is set by an external source, Equation (14) can be solved only on the magnetic surface, where the equality $\omega = \Omega_{TN}(x^1)$ holds. We refer to this surface as toroidal and denote it by x^1_{TN}. Name the eigenfunctions $T_N(x^1, x^3)$ of (14), (11) the toroidal eigenfunctions. To normalize these functions, use the following condition: the maximum value of $T_N(x^1, x^3)$ along the field line is equal to 1.

In the latter extreme case, κ=0, the solution of Equation (9) is determined by the addend

$$\hat{L}_P(\omega)\Phi = 0 \tag{15}$$

with boundary condition (11). The eigenfrequencies in this equation Ω_{PN} are referred to as poloidal. They depend on the radial coordinate x^1 as on a parameter. At the wave frequency ω, Equation (15) can be solved only on the magnetic surface, where the equality $\omega = \Omega_{PN}(x^1)$ holds. We will denote this surface the poloidal surface and designate it as x^1_{PN}.

Call the eigenfunctions $P_N(x^1, x^3)$ of (15), (11) poloi dal. To normalize these functions, use the following condition: the maximum value of $P_N(x^1, x^3)$ along the field line is equal to 1.

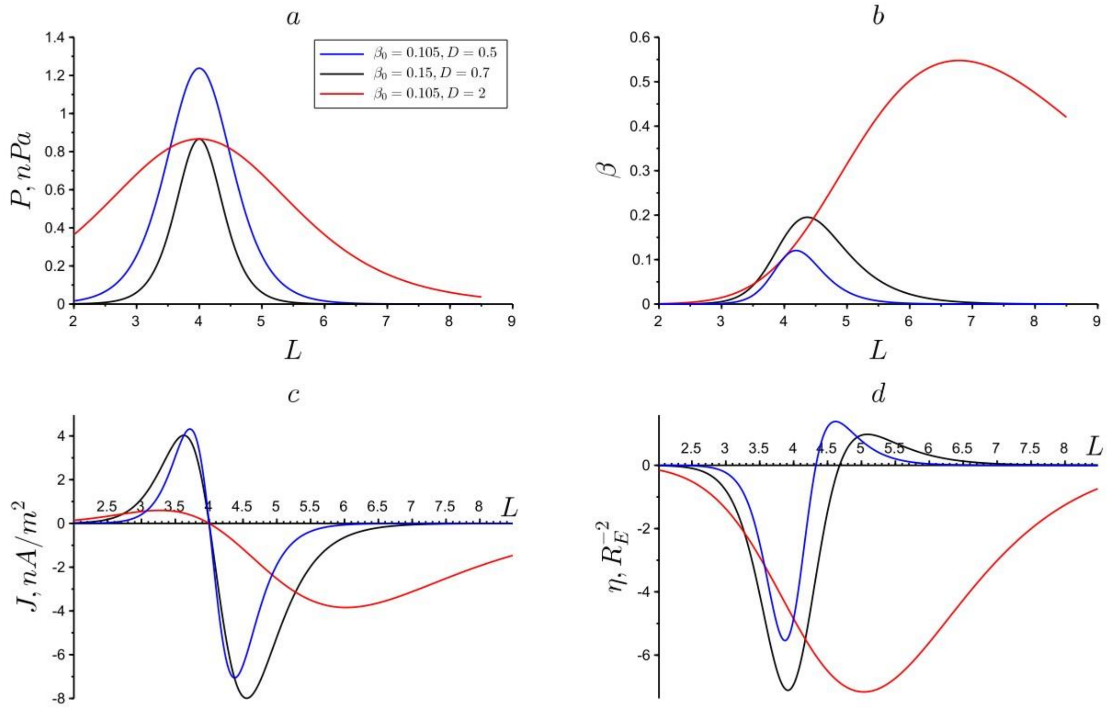

Figure 1. Plasma pressure profiles P (*a*), parameter β (*b*), ring current J (*c*), and η (*d*)

In most of the magnetosphere (except for the regions near the plasmapause whose existence we ignore), the eigenfrequencies Ω_{PN} and Ω_{TN} decrease with increasing L shell. Numerical solutions of (14) and (15) show that in the model with cold plasma β=0 the poloidal frequency is always lower than the toroidal one: $x^1_{PN} < x^1_{TN}$ [Cummings et al., 1969; Leonovich, Mazur, 1993].

Radial profiles of the eigenfrequencies in models 1–3 are presented in Figure 2. It is apparent that when taking into account the plasma pressure and its gradient, both variants are possible: $\Omega_P<\Omega_T$ and $\Omega_P>\Omega_T$. Hence, when considering the final pressure, the poloidal surface, compared to the toroidal one, can be both closer to Earth and farther from it. Of particular attention is the minimum of the poloidal frequency in model 1 with β_0=0.105, D=0.5, caused by the ring current due to a negative pressure gradient. In all the models considered, squares of the eigenfrequencies are positive, i.e. the assumption that there is no hydromagnetic instability (ballooning or permutational) is valid.

5. VARIATION IN THE SPATIAL STRUCTURE OF ALFVÉN WAVES WITH DISTANCE IN THE TRANSVERSE WKB APPROXIMATION

Now let us tackle eigenproblem (9) with respect to $\kappa^2 = k_1^2/k_2^2$ at a fixed wave frequency [Leonovich, Mazur, 1993]. The numerical calculations are made for two frequency values: ω=0.033 and 0.018 rad/s. We deal only with the fundamental harmonic (N=1). For each given frequency, the value κ is a function of the radial coordinate x^1. The wave propagation region (transparent region) corresponds to κ^2>0. On the poloidal surface x^1_{PN}, the eigenvalue κ^2=0; near the toroidal surface x^1_{TN}, $\kappa^2 \to \infty$. The corresponding points along the radial coordinate are called reflection and resonance points respectively. On the poloidal and toroidal surfaces, the eigenfunction Φ_N of Equation (9) coincides with the poloidal P_N and toroidal T_N eigenfunctions of Equations (14) and (15). At intermediate values of κ, i.e. on the magnetic surfaces lying in the gap between the poloidal and toroidal surfaces, the longitudinal structure of harmonics Φ_N gradually changes from poloidal to toroidal.

5.1. Model 1

The behavior of $\kappa^2(x^1)$ of model 1 at a frequency ω =0.033 rad/ s is illustrated in Figure 3. The transparent region is seen to be limited by the points of reflection (poloidal surface x^1_{PN}) and resonance (toroidal surface x^1_{TN}), with $x^1_{PN} < x^1_{TN}$.

The transparent region (κ^2>0) in this case becomes wider by ~0.35 R_E (see Figure 2). In most of the transparent region, the $\kappa^2(x^1)$ dependence is more smooth than in cold plasma [Leonovich, Mazur, 1993]. This leads to some difference in the longitudinal structure of harmonics Φ_N: the structure changes from poloidal to toroidal in a more uneven way (Figure 4, *a*, *b*).

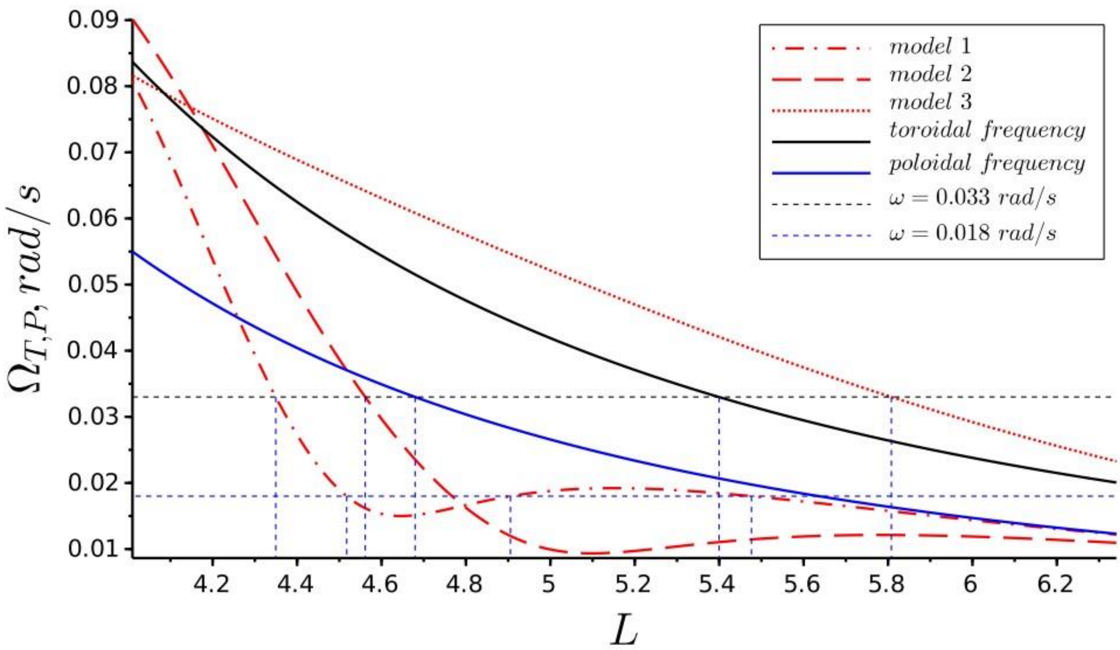

Figure 2. Toroidal and poloidal eigenfrequencies $\Omega_{TN,PN}$ as a function of L for the fundamental harmonic of the standing wave (N =1). In cold plasma (β=0), the poloidal frequency (blue curve) is always lower than the toroidal one (black curve). In models 1 and 2, the poloidal frequency is also lower than the toroidal one (red dash-dot and dashed lines). In model 3, the poloidal frequency (red dotted line) becomes higher than the toroidal one. Horizontal and vertical dashed lines indicate the position of the frequencies Ω=0.033 and Ω=0.018 rad/s with respect to their coordinates

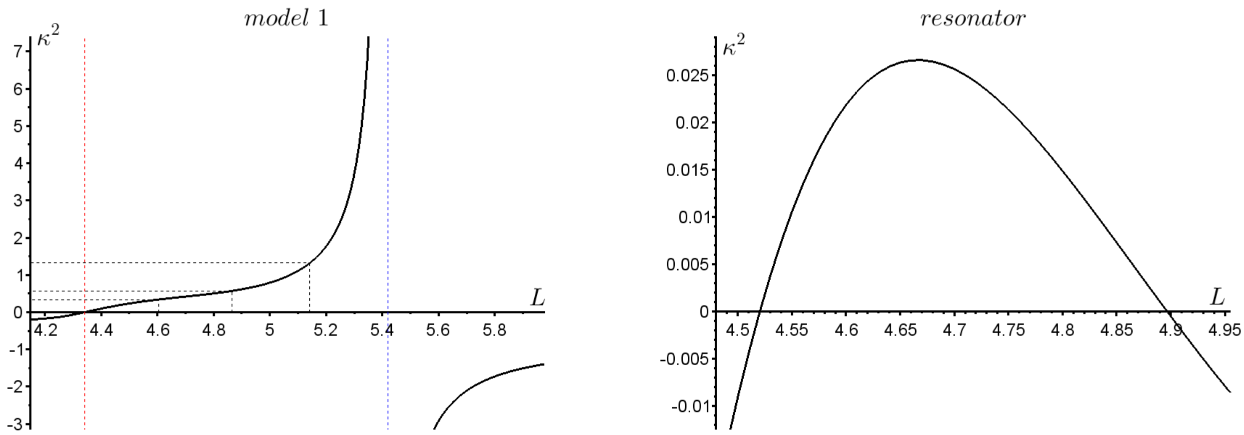

Figure 3. κ^2 as a function of L at β_0=0.105 and D=0.5 with wave frequencies ω=0.033 (left) and ω=0.018 (right) rad/s for model 1

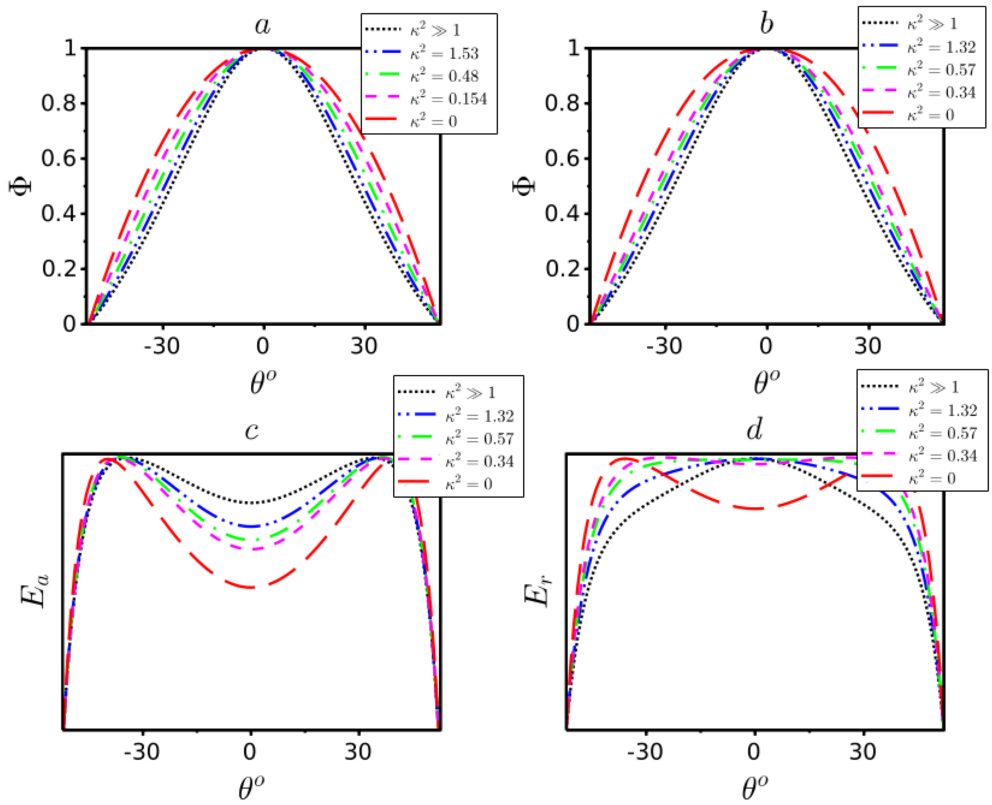

Figure 4. The Alfvén wave harmonic structure (panel *a*) as a function of the magnetic latitude θ between magnetic shells in the transparent region of wave κ^2>0 for the cold plasma model (panel *a*) and model 1 (panel *b*) at a frequency ω=0.033 rad/s. Azimuthal (panel *b*) and radial (panel *d*) electric field components on the poloidal (red dashed line) and toroidal (black dotted line) shells of model 1

Referring to Figure 4, *b* (red dashed curve), in model 1 the azimuthal electric field component E_a near the toroidal surface differs from the poloidal one in that it gradually forms a deep minimum. The radial field component E_r has a minimum near the poloidal surface, and a maximum on the toroidal surface (Figure 4, *d*, black dotted line). Of special interest is the behavior of the compression component of the magnetic field $B_{||}$: it sharply nonmonotonically depends on the longitudinal coordinate, although it does not change sign (see Figure 5).

Interesting features arise at a frequency ω=0.018 rad/s (see Figure 3): the dependence $\kappa^2(x^1)$ becomes nonmonotonic, due to which an additional transparent region with a width of ~0.5 R_E, bounded by two transition points with κ^2=0, appears. In this region, the Alfvén wave forms a transverse resonator, where the mode is stationary along the field line and across magnetic shells, but propagates along the azimuth [Klimushkin et al., 2004].

5.2. Model 2

If in cold plasma and in model 1 the poloidal surface was located closer to Earth than the toroidal one, in model 2 the situation becomes the opposite: the poloidal surface is located farther from Earth (see Figure 2, red dashed line). The cause is a negative gradient of plasma pressure and a rather small value of β. The behavior of the harmonics Φ_N and the components of electric and magnetic fields, therefore, differs slightly from model 1 in this case. A characteristic difference for this model is, however, the change of the sign of the longitudinal magnetic field component along the field line in the vicinity of the equator near the poloidal surface (Figure 5, red dashed curve).

5.3. Model 3

As for magnetosphere model 3, its maximum value β is much greater than that in models 1 and 2. In this case, the poloidal surface is located farther than the toroidal one. The transverse behavior of κ^2 looks, therefore, as shown in Figure 6, left. Due to the large curvature of field lines near the equator and the large value of β, an Alfvén wave opacity area appears near the poloidal surface along field lines in the equatorial part of the magnetosphere (Figure 6, right).

Because of this, the wave processes in the Northern and Southern hemispheres are weakly interconnected. The possibility of formation of an opacity area along the field line for a poloidal Alfvén wave was first shown in [Mager et al., 2009] and then confirmed in [Mazur et al., 2012; Leonovich, Kozlov, 2013]. A consequence of the nonmonotonic behavior of the potential Φ along the field line is the nonmonotonic behavior of the transverse components of the magnetic field of the wave (Figure 7, *a*–*d*): these components have not one (equatorial) node, as in cold plasma and models 1 and 2, but three nodes. Attention is also drawn to the sharp peak of the longitudinal component of the magnetic field of the wave near the equator and the change of its sign along the field line (Figure 5).

CONCLUSIONS

Here are the main results we obtained.

1. We have addressed an eigenproblem with respect to the wave frequency for the equation describing the wave structure of Alfvén modes [Klimushkin et al., 2004] in two extreme cases: $\kappa\to\infty$ and $\kappa\to 0$ corresponding to toroidal and poloidal modes. A pressure increase on this magnetic shell was shown to cause the poloidal frequency to increase, while an increase in the pressure gradient contributes to its decrease.

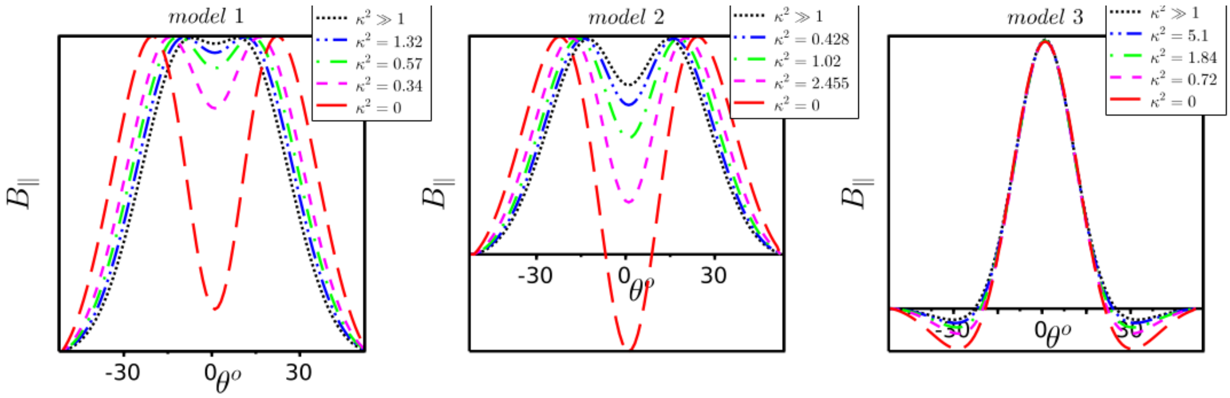

Figure 5. Changes of the parallel magnetic field component structure in θ for models 1–3 at ω=0.033 rad/s

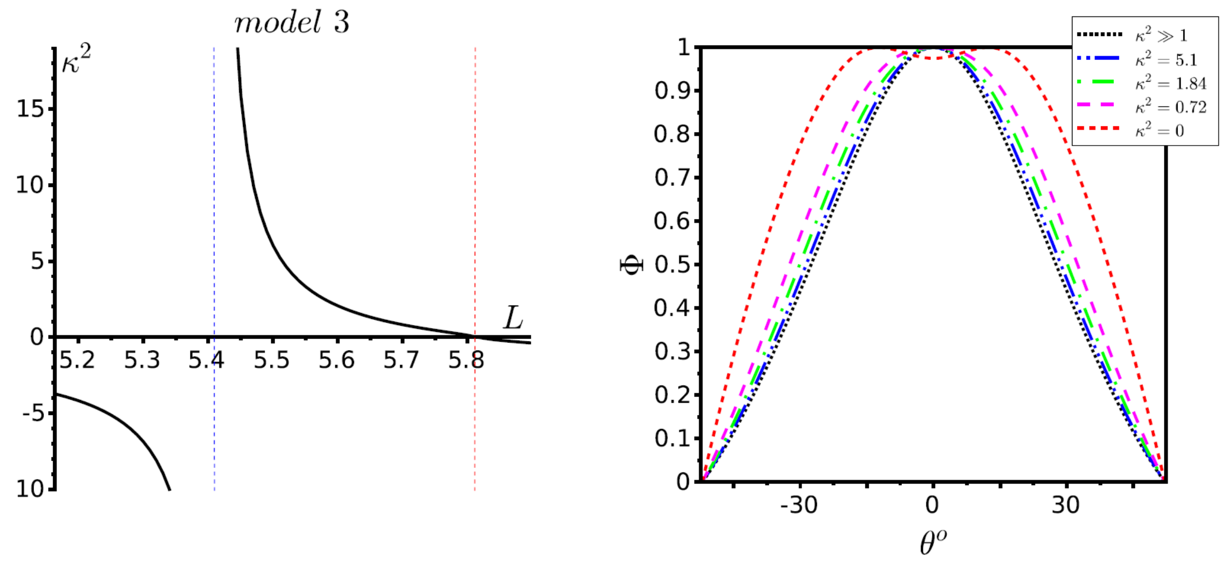

Figure 6. κ^2 as a function of *L* (left) and the Alfvén wave harmonic structure (right) versus the magnetic latitude θ between magnetic shells in the transparent region of wave $\kappa^2>0$ at $\beta_0=0.105$ and *D*=2 for model 3 with a fixed wave frequency ω=0.033 rad/s

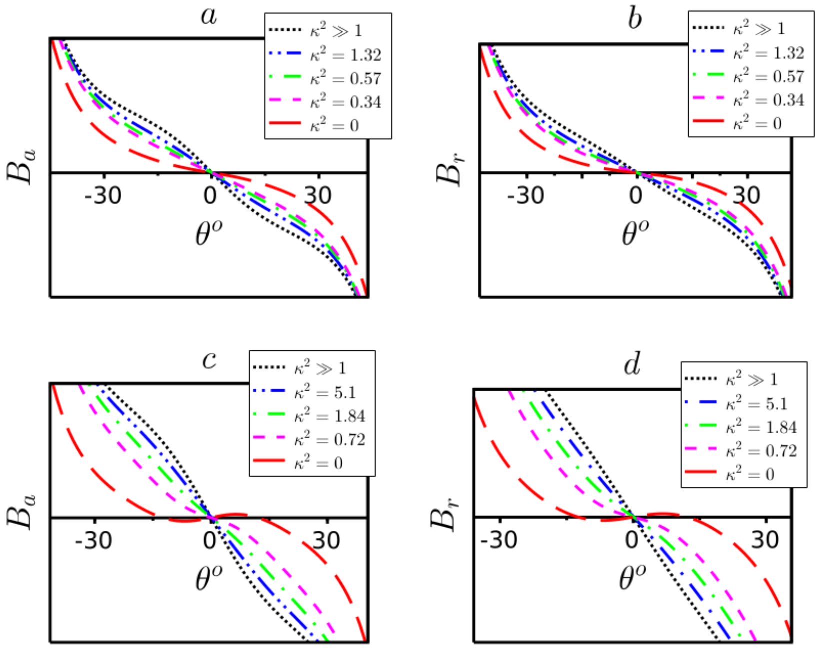

Figure 7. Changes of the structure of azimuthal and radial magnetic field components in θ for models 1 (panels *a*, *b*) and 3 (panels *c*, *d*)

2. We have solved the eigenproblem with respect to κ at a fixed wave frequency. We have identified the transparent region of the wave, where $\kappa^2>0$. When moving across magnetic shells from the poloidal surface (where $\kappa = 0$) to the toroidal one (where $\kappa=\infty$), the wave changes its polarization from poloidal to toroidal. We have determined radial coordinates of the wave transition point (the poloidal surface where $\kappa=0$) in different magnetosphere models. We have shown that in models 1 and 2, the poloidal surface is located closer to Earth than the toroidal one; in the former case, the transparent region is several times wider than in the latter one. In model 3, the poloidal surface is located farther from Earth than the toroidal one.

3. We have established that with a sharp transverse localization of the equilibrium current the poloidal frequency has a minimum at a certain value of L. There may be a transverse Alfvén wave resonator near the minimum of the function $\Omega_{PN}(L)$. We have examined the longitudinal structure of the wave on different magnetic shells inside the transparent region corresponding to different wave polarization. Due to the large curvature of field lines and the parameter β in model 3, a minimum in the fundamental Alfvén wave harmonic appears in the vicinity of the equator [Mager et al., 2009]. The magnetic field of the wave in model 3 has three nodes, not one as in models 1 and 2. In models 2 and 3, the longitudinal component of the wave magnetic field changes sign along the field line.

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